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Numerical calculation method of various differential equations

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ABSTRACT : *the differential equation by ordinary differential equation, a differential equations and partial differential equations, different equation algorithm. In this paper, the above three kinds of equations as an example, analyzes the different characteristics of different calculation methods. On this basis, the optimization calculation process of various types of equations, hope to be able to improve the computational efficiency of the equation purpose.*

KEYWORDS: *ordinary differential equations; differential equations; partial differential equations*

I. INTRODUCTION:

Differential equations are the main computational models used in engineering calculation and research fields. On the basis of ensuring the accurate solution, improving the computational efficiency of the equation is the focus in all fields. When calculating various types of differential equations, it is difficult to obtain the expression of the solution, which makes the optimization method become very important.

Differential equation classification and solution: The differential equation is the relation between the independent variable, the unknown function and the derivative of the unknown function. According to the number of independent variables, differential equations can be divided into ordinary differential equations and partial differential equations: (1) Ordinary differential equations: When the number of independent variables in a differential equation is 1, the equation can be called ordinary differential equations. What's more, Traditional differential equations must be calculated after finding interpolation points. Based on the concept of deviation, the first-order or second-order deviation method is used to solve the ordinary differential equation, which can effectively solve the difficulties caused by the process of solving the interpolation points, thus reducing the difficulty of equation calculation and improving the calculation efficiency[1].（2）Differential Equations: Whether the solution to a system of differential equations will be stable or not depends on the characteristics of the integrand. At present, the calculation methods mainly include single step method and multi - cloth method. The Euler calculation method is a kind of one-step problem solving method of differential equations, which has certain representativeness, but it is difficult to solve and the efficiency is low. Explicit and implicit solutions are two common forms of solutions of differential equations, and the latter is more accurate. In a word, Optimizing the method of differential equations is the main way to obtain implicit solutions [2].（3）Partial differential equations: When the number of independent variables for the differential equation is 2 or more, this equation can be called partial differential equations. Only a small part can be calculated by the exact solution among partial differential equations. Most equations are extremely difficult to calculate, the results obtained can only approach the exact value and the accuracy of the calculation needs to be improved. However, using the finite-time method to discretize the continuous problem can solve the above problem [3].

II. NUMERICAL CALCULATION OF ORDINARY DIFFERENTIAL EQUATIONS

The definition of ordinary differential equations

Suppose multivariate function $f(x_1, x_2, x_3, \cdots x_n)$ exists. $\left(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \cdots, x_n^{(k)}\right)$ is the kth differential result of $f(x_1, x_2, x_3, \cdots x_n)$, then the second-order deviation formula can be obtained. Considering the order of the functions has no effect on the higher-order partial differentiation, it is not necessary to consider the order of the functions in the calculation process. When i and j are not equal, the second-order deviation can be divided into four major categories. However, when the value of i is equal to the value of j, the second-order deviation can only be divided into three types.

Deviation derivation: In order to improve the efficiency and accuracy of the function calculation, the concept of deviation can be introduced into the calculation process to make it approximate calculation instead of the derivative：

There are ordinary differential equations as follows： $\frac{dy}{dx}=f(x,y)$, $y_0=x_0$. In this equation, y_1 and y_2 of $f(x,y)$ satisfy Lipschitz condition. According to the idea of deviation, the formula of the deviation method can be derived, and the explicit and implicit Euler method is obtained, namely $y_{k+1}=y_k+hf\left(x_k,y_k\right)$ and $y_{k+1}=y_k+hf\left(x_{k+1},y_{k+1}\right)$. It can be seen from the formula that the above derivation method has no partial difference, so the Euler method can be regarded as zero order deviation method.

Weighted average deviation: In the traditional calculation of ordinary differential equations, the Euler method should be averaged to form a trapezoidal method for solving [4]. Besides, wighted methods also have an average effect. Multiple ordinary differential equations can be combined to achieve the purpose of the average Euler method through weighted methods. Take the zero-order weighted average deviation method as an example: Set the weight to s , $0<s<1$, and combine the zero-order weighted formula to get the following formula: $y_{k+1}=y_k+h\left[sf(x_k,y_k)+(1-s)f(x_{k+1},y_{k+1})\right]$. If $s=0.5$, the above formula can be regarded as the representative formula of the trapezoid method.

Calculating the stability: We have the equation $y'=\lambda y,\lambda<0$. At this point, the experimental method can be used to evaluate the stability of the calculation method. By introducing λ into the trapezoidal method representative formula, the following formula can be obtained:

$$y_{k+1}=y_k+h\lambda y_k+1/2h(\lambda y_k+1-\lambda y_k)=y_k+h^*y_k+1/2h^*\left(y_{k+1}-y_k\right)$$

You can get the stability condition of the equation after the migration and the solution. The final solution shows that when the stable region is $1/2+1/h^{**}<1$ the weighted value of s is $\left(0,1/2+1/h^{**}\right)$. In this case, the solution method is stable. When the stable region is $1/2+1/h^{**}\geq 1$, the weighted value of s is $\left(0,1\right)$. At this point, the method is also absolutely stable. It can be considered that the above calculation method is more stable.

Calculation process: Let $y=-1-x+2e^x$. Using the trapezoidal method and weighted average method to solve, the results shown in Table 1:

Table 1 **Different calculation results for different methods**

x	Trapezoidal method	Weighted average method	Explicit Euler	Implicit Euler	Exact solution
0.1	1.110526	1.110333	1.100000	1.122222	1.110342
0.3	1.400393	1.399686	1.362000	1.443483	1.399718

Using first-order deviation method to solve, you can get the calculation formula of each formula. Summarizing the calculation results of the calculation format, it can be found that, compared with the explicit and implicit Euler methods, the first-order deviation method is used to solve the problem with higher accuracy. Compared with the weighted average method and the calculation accuracy of the trapezoidal method, we can see that as long as the former has the right weight, the accuracy of the result is generally higher than the latter.

III. NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

Finite element method solving steps and calculation method

Finite element method solving steps: The steps to solve the partial differential method in Finite element method of are as follows: （1）The original equation is divided into different variational forms.（2）Select the shape and

split the area.（3）Establish basis functions to form the corresponding space.（4）Confirm discrete
5）Solve the function.

Finite element method calculation process

With partial differential equations are as follows：

$$-\alpha\Delta u+\beta u=s,x=\Omega$$

$$U=d,x\in D$$

$$\alpha\frac{\partial u}{\partial v}+ru=r,x\in F$$

Where，$\frac{\partial u}{\partial v}$ is the derivative, Ω represents the bounded region，α、β、t、s and d are given constant. Using triangulation method to divide the region, we can get the following basis function. The formula is as follows: $\varphi_1=a_1+b_1x+ey$. The formula satisfies $\varphi_1(x_1,y)=1,\varphi_1(x_1,y_1)=0,\varphi_1(x_2,y_2)=0$ and we can get the expression of the basis function by taking it to $\varphi_1=a_1+b_1x+ey$.

Finite difference method and calculation method : Finite difference method solving steps: The steps of using finite difference method to solve differential method are as follows: （1）Find the solution area and divide the grid.（2）Determine the difference format according to the type of equation to be calculated.（3）Analyze the convergence and stability of the equation.

Finite difference method calculation method

There are partial differential equations as follows:

$$U_2+\Delta 2u=|u|y\quad (x,t)\in(0,1)$$

$$U(x,0)=u,\quad (x)\quad \mathrm{x}\in(0,1)$$

$$\mathrm{U}=0,\frac{\partial u}{\partial v}=0,t\in(0,1],x=0,x=1$$

Where, $p>1$, Ω represents a bounded area. The time step is set to $\Delta t=1/N$ and the spatial step size $h=1/M$ after aliasing the spatial region $[0,1]$. is divided into equal M parts. This is grid $I=\{x,m=0,1,2,L,M\}$, and φ represents the grid function. Based on the above conditions, the corresponding difference quotient and the finite difference format of the function can be obtained to judge the convergence of the format.

IV. NUMERICAL CALCULATION OF DIFFERENTIAL EQUATIONS

Calculation method

Euler method: Euler method belongs to the main calculation method of ordinary differential equation and the formula is fixed. In the formula, there are some items such as node sequence, step length and known function. In which, the node sequence is represented by K , and the step length is expressed by h , and $f(x,y)$ is used for the known function. Using the two-step Euler method, the second-order algebraic accuracy can be achieved. The equation is as follows：$Y_{k+1}=Y_{k+1}+2hf(x,y)$。Using the improved Euler method, we can get the implicit formula (system of equations) of Euler's method:

$$Z_k=Y_{k+1}+hf(x_{k-1},y_{k-1})$$

$$Y_k=Y_{k+1}+0.5hf(x_{k-1},y_{k-1})+f(x_k,y_k)$$

The above formula has the characteristics of forecasting-correcting, which called Runge-Kutta method. This method has a variety of forms, and the 4th order Runge-kutta method is the most widely used. From our point of

view, we can use Taylor series to derive the 4th order Runge-Kutta formula.
If the equation has good smoothness, the numerical accuracy obtained will be generally higher by using this method. Nevertheless, if the equation has poor smoothness, the numerical accuracy obtained will be generally lower by using this method.

Improved Euler method: In the case of poor smoothness of the equation, the improved Euler method can be used to solve the problem to further improve the accuracy of the calculation. The 2nd order improvement Euler method is as follows：

$$K_1 = hf\left(x_k, y_k\right)$$
$$K_2 = hf\,(x_k + 0.5hy_k + 0.5k_1)$$
$$K_3 = hf\left(x_k + 0.5hy_k + 0.5k_2\right)$$
$$K_4 = hf\left(x_k + hy_k + k_3\right)$$
$$Y_{k+1} = y_k + (K_1 + K_2 + K_3 + K_4)/6$$

The above methods are single-step method. The initial calculation of differential equations usually applies the one-step method. After using the single-step method to get the corresponding point, it can use the multi-step method to calculate. Runge-Kutta method belongs to the main form of single-step method, and the accuracy can be guaranteed. Therefore, the Runge-Kutta method can be used as the primary method for initial calculation. Finally, we can use the Simpson formula to establish the corresponding recursion formula. The recursion formula is as follows：

$$Z_k = Y_{k-3} + 3(y)Y_{k-1} - Y_{k-2}$$
$$Y_k = Y_{k-2} + hf\left(X_{k-2}, Y_{k-2}\right) + 4f\left(X_{k-1}, Y_{k-1}\right) + f\left(x_k, y_k\right)/3$$

Depending on the number of recursion formulas applied, the function of the formula is not the same.

4th order Adams extrapolation to interpolation method : When the number of calculations is only 1, the recurrence formula can be used as a forecast - correction method. When the calculation is more frequent, the formula can play the function of iterative calculation. It should be noted that if the number of iterations is too high, the accuracy of the calculation results can be easily affected. Therefore, the number of iterations is about 2 times is more appropriate. In addition to the recursion formula, the 4th order Adams extrapolation to the interpolation method, also belongs to the forecast - a correction method. The formula is as follows：

$$Z_k = Y_{k-1} + h(55f_{k-1} - 59f_{k-2} + 37f_{k-3} - 9f_{k-4})/24$$
$$Y_k = Y_{k-2} + h(9f_k + 19f_{k-1} - 5f_{k-2} - 9f_{k-1})/24$$

The calculation accuracy of this formula is greatly affected by the step size. As the step size decreases, the accuracy of the calculation results can be improved.

Calculation and analysis

There are differential equations as follows：

$$y' = 4^{\alpha\beta x} = 0.5y$$
$$Y\left(0\right) = 2$$

The equation is： $y\left(x\right) = 4/1.3(e^{\alpha\beta x} - e^{-\alpha\beta x}) + 2^{\alpha\beta x}$

After calculation, we can get the step. During the calculation, it is found that the Runge-Kutta method has higher computational accuracy when $h = 0.1$, which is superior to the 4th order Adams method. In addition, the Simpson method has the worst accuracy. Accordingly, when the step length is 0.1, it's more suitable to use the Runge-Katta method.

Calculation summary: Comparing the accuracy of calculation results of each calculation method shows that the implicit solution of differential equation is more stable than the explicit solution.

We can draw the conclusion that the results are more accurate through using the Runge-Kutta method, the 4th order Adams method and the Simpson method. However, the Runge-Kutta method is more accurate than other methods, so this method can be used to calculate differential equations.

V. CONCLUSION

Through the study on the calculation methods of various kinds of differential equations, it is found that each calculation method has its scope of application, and the accuracy of the solution also has some differences. In the process of equation calculation, the application of different algorithms should be applied to make the accuracy of the calculation result to a greater degree.

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